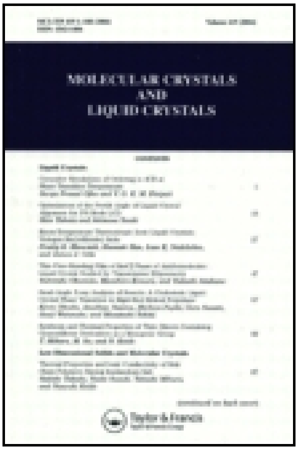

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PHASE MODULATION TIME DEPENDENCE MEASUREMENT

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PHASE MODULATION TIME DEPENDENCE MEASUREMENT

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Phase modulation using liquid crystals has wide-ranging applications including correlation [1] and telecommunications [2,3]. At present, relatively slow (order 5ms) nematic cells demonstrate the advantage of continuous phase modulation but are sensitive to polarisation of the input light [4,5]. The alternative is to use faster ferroelectric cells which exhibit polarisation insensitivity but show only binary phase modulation [2]. This work demonstrates a novel technique for measuring the phase response time, which allows evaluation of different cell topologies. Rise and fall times are calculated in the Fourier plane from a wave-front splitting interferometer. Phase modulation measurements of in-house cells have been made for planar and hybrid aligned liquid crystal cells using nematic liquid crystals. The results demonstrate the capabilities of in-house cells, and also show agreement with theoretical predictions of cell behaviour.

Keywords: dynamic; fourier; liquid crystal; phase modulation

1. INTRODUCTION

Phase modulation is recognised to depend on the polarisation state of the incident light in two ways. Firstly, the electric field output state (E_{OUT}) may differ from the input state (E_{IN}), and secondly the phase may depend on the input state. Ideally, phase modulation (ϕ_M) would be independent of the input state, and preserve the polarisation state of the incident light. This may be represented in the Jones [6] formalism (1).

$$E_{OUT} = e^{j\phi_M} E_{IN} \tag{1}$$

A worst case phase modulator therefore has phase and output polarisation dependent on the input polarisation state. Current phase modulators tend

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to modify the polarisation state of the light, but achieve useful phase modulation (2) if the input polarisation state is restricted (E_{CONST}), for example by polarisers. However, for general input states the output intensity may vary. If the input polarisation can be defined these devices work in a controlled manner.

$$E_{OUT} = e^{j\phi_M} E_{CONST} \tag{2}$$

Methods of measuring the phase modulation capability of a liquid crystal cell are in demand as interest in phase modulation increases [1,3]. Both the static and dynamic properties are important when comparing different cell topologies.

In this paper, an extension of the phase measurement method [7] is made to measure firstly static polarisation properties of nematic liquid crystal cells. Accurate theoretical predictions of the polarisation dependence can be made using Fourier and Jones analysis.

This method also allows the time for a phase change to be measured. Essentially, a photodiode is used to record the rate at which the phase modulation takes place. The phase modulation is calculated, then the time taken for the phase to change from 10 to 90 percent is measured. Traditional measurements of switching times (t_{ON} and t_{OFF}), defined as the time for intensity output of a cell between crossed polarisers to change from 10 to 90 percent, are not appropriate as a phase change of more than π radians will lead to a maxima or minima in the intensity. Measurements are important to demonstrate cell behaviour, as numerical simulation of twisted nematic cells demonstrate the importance of flow terms, a factor ignored in many analyses.

2. EXPERIMENTAL METHOD

The experiment used a standard '4f' system (Fig. 1) including a mask with two slots (Fig. 2) placed behind a liquid crystal cell. The cell requires only 2 pixels, arranged so that one pixel is behind each of the slots. A polariser and half wave plate were used to vary the incident polarisation.

In this investigation, one pixel is maintained with zero voltage across it, whilst the other has a variable voltage across it. The resulting interference pattern (Fig. 3) on the photodiode or CCD can be used to reveal the phase information.

This method of measurement is good because variation of phase modulation with time can be measured for in-plane and out-of-plane changes to birefringent media using the same equipment. Cell topologies (Fig. 4) were based on nematic liquid crystals.

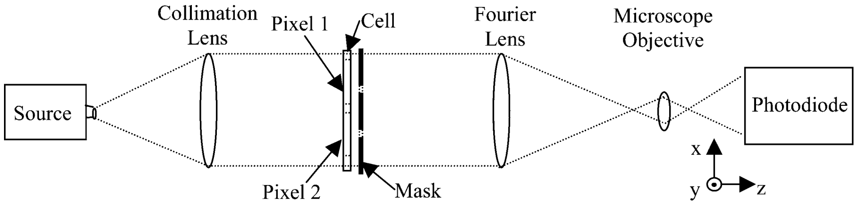

FIGURE 1 Experimental equipment.

All cells, with the exception of one, were close to 5 μm thickness. One 10 μm thick was used for measurement of fall time for a phase of 2π. Homeotropic alignment was achieved using ZLI3344, and planar using AM4276 for a pre-tilt of 2–3 degrees, and SE3150 for a pre-tilt of about 8 degrees. Cells were filled with nematic liquid crystal mixture Blo48 (E44). In this document, cells are identified by the alignment type.

3. MEASUREMENT ANALYSIS

The interference pattern in the far field is calculated using Fourier transforms (3), where E_{OUT1} and E_{OUT2} are the output states from the liquid crystal cell pixels. Some phase factors not dependent on the liquid crystal have been excluded. This analysis improves calculations [7] where only the output magnitude and phase difference are considered.

$$\begin{aligned} F(u) &= \int_{-x_0-d}^{-x_0+d} E_{OUT1} e^{2\pi jxu} \cdot dx + \int_{x_0-d}^{x_0+d} E_{OUT2} e^{2\pi jxu} \cdot dx \\ &= 2d \sin c(2\pi ud)(E_{OUT1} e^{-2\pi ux_0} + E_{OUT2} e^{2\pi ux_0}) \end{aligned} \tag{3}$$

Where x_0 and d are mask parameters, and u is the spatial frequency. The

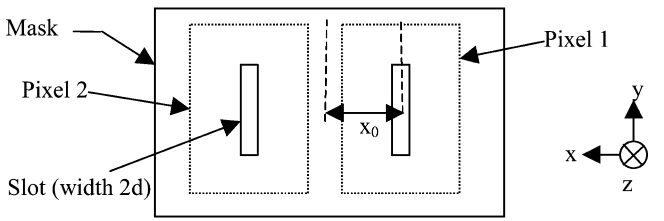

FIGURE 2 Mask description.

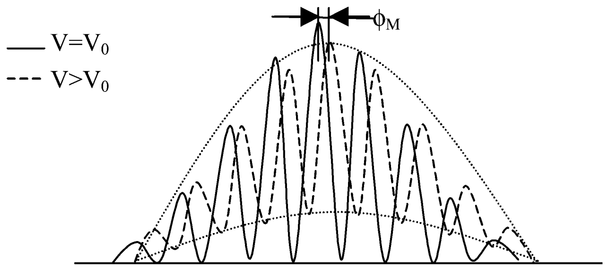

FIGURE 3 Fourier plane interference pattern for two different voltages applied.

intensity (4) is given by the Hermitian scalar product [6], equivalent to the magnitude squared of the electric field in the far field.

$$Intensity = 4d^2 \sin c^2(2\pi ud)(2 + \gamma \cos(4\pi x_0 u + \phi_M)) \tag{4}$$

$$\text{where } \phi_M = -\tan^{-1}\frac{\delta}{\beta} \text{ and } \gamma = \beta\cos(\phi_M) + \delta\sin(\phi_M) \tag{5}$$

$$\text{and } \beta = E^*_{OUT1}E_{OUT2} + E^*_{OUT2}E_{OUT1} \text{ and } \delta = j(E^*_{OUT1}E_{OUT2} - E^*_{OUT2}E_{OUT1}) \tag{6}$$

The output state, which is generally elliptical, is found by concatenating the Jones matrices [6] describing each component in the usual way. The Jones matrix for a cell at angle θ with retardation Γ and phase factor ξ (7) may be used, where n_e and n_o are the extraordinary and ordinary refractive indices respectfully.

$$J_{CELL} = e^{j\xi/2}\begin{bmatrix} A & B \\ B & A^* \end{bmatrix} \tag{7}$$

where $A = \cos\frac{\Gamma}{2} - j\sin\frac{\Gamma}{2}\cos 2\theta,\ B = -j\sin\frac{\Gamma}{2}\sin 2\theta,\ \Gamma = \frac{2\pi(n_e - n_o)d}{\lambda}$ and $\xi = \frac{2\pi(n_e + n_o)d}{\lambda}$

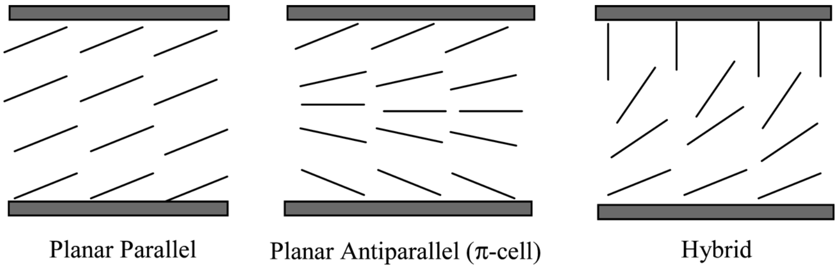

FIGURE 4 Cell topologies measured.

Intensity modulating liquid crystal cells between crossed polarisers are dynamically characterised by the time for intensity to change from 10 to 90 degrees. Dynamic properties of a phase modulating cell might be inferred using this configuration, but the method described in this paper has the advantage that phase is measured directly. The proposed method is also independent of the intensity of the light source, and polarisation dependence can be evaluated. In general, the Jones expression for the output state (8) from a birefringent cell (7) is related to the input polarisation, of complex components V_X and V_Y.

$$E_{OUT} = J_{CELL}E_{IN} = J_{CELL}\begin{bmatrix} V_X \\ V_Y \end{bmatrix} = J_{CELL}\begin{bmatrix} E_X \\ E_Y e^{j\phi} \end{bmatrix} \tag{8}$$

For a loss and amplification free media the phase modulation (5) can be calculated (9). Subscripts indicate the pixel concerned. Thus the measured phase of a cell is a complicated combination (10) of the retardation, optical axis rotation and input polarisation.

$$\begin{aligned} \beta &= 2K\cos(\xi_2/2 - \xi_1/2) + 2L\sin(\xi_2/2 - \xi_1/2) \\ \delta &= -2K\sin(\xi_2/2 - \xi_1/2) + 2L\cos(\xi_2/2 - \xi_1/2) \end{aligned} \tag{9}$$

Where

$$\begin{aligned} K &= 2\cos(\Gamma_1/2)\cos(\Gamma_2/2) + 2\sin(\Gamma_1/2)\sin(\Gamma_2/2)\cos(2\theta_2 - 2\theta_1) \\ L &= -2(L_{\square}(E_X^2 - E_Y^2) + 2(L_{\triangle}\sin\phi + L_{\diamond}\cos\phi)E_XE_Y) \\ L_{\square} &= \sin(\Gamma_1/2)\cos 2\theta_1\cos(\Gamma_2/2) - \cos(\Gamma_1/2)\sin(\Gamma_2/2)\cos 2\theta_2 \\ L_{\triangle} &= \sin(\Gamma_1/2)\sin 2\theta_1\cos(\Gamma_2/2) - \sin(\Gamma_2/2)\sin 2\theta_2\cos(\Gamma_1/2) \\ L_{\diamond} &= \sin(\Gamma_2/2)\sin(\Gamma_1/2)\sin(2\theta_2 - 2\theta_1) \end{aligned} \tag{10}$$

A liquid crystal cell that undergoes only a change in effective birefringence demonstrates that the measured phase is polarisation dependent (11). It is interesting to note the relative phase of the orthogonal components of the input polarisation is unimportant.

$$\phi_M = -\tan^{-1}\left(\frac{-\sin(\Gamma_1/2 - \Gamma_2/2 + \xi_1 - \xi_2)E_X^2}{\cos(\Gamma_1/2 - \Gamma_2/2 + \xi_1 - \xi_2)E_X^2 + E_Y^2}\right) \tag{11}$$

4. DYNAMIC ANALYSIS

Rise time (12) and fall time (13) have been analysed in the past for twisted nematic cells [8], which can be applied to non-twist cells when the twist of the cell is zero.

$$\tau_{ON} \propto \frac{\gamma_1 d^2}{\varepsilon_0 \Delta\varepsilon (V^2 - V_0^2)} \tag{12}$$

The cell has thickness (t), liquid crystal properties rotational viscosity (γ) and dielectric anisotropy ($\Delta\varepsilon$).

$$\tau_{OFF} \propto \frac{\gamma_1 d^2}{\varepsilon_0 \Delta\varepsilon V_0^2} \tag{13}$$

These equations ignore pre-tilt and flow terms. An alternative (14) was also given, derived originally by Jakeman and Raynes.

$$\tau_{OFF} = \frac{\eta d^2}{K\pi^2} \tag{14}$$

Pi-cells (or OCB – Optical Compensated Birefringence cells) have been suggested to improve switching times if the cell is kept in the bend state, and prevented from entering the splay state [9]. Some measurements were taken on antiparallel cells to observe fall time when the voltage was kept above a critical level where the lowest energy state changes from splay to bend.

5. STATIC RESULTS AND DISCUSSION

Phase modulation measured dependence on polarisation and applied voltage (Fig. 5) of a planar cell with antiparallel alignment is in close agreement with theoretical predictions. The phase measured is continuous, but has been plotted in a range $-\pi$ to $+\pi$. Thus for 0 to 45 degrees every crossing of the x axis indicates a phase modulation of π. This cell therefore achieves a phase change of almost 4π.

The theoretical phase modulation polarisation dependence was calculated (11) assuming that the measured phase gives an accurate estimate of effective refractive index dependence on voltage, when the polarisation was aligned to the long axis of the molecules. This assumption appears to be reasonable, as the analytical and measured curves are in good agreement. This process can be carried out with accuracy for any cell. Whilst all planar cells (parallel and antiparallel) were found to have similar static phase and polarisation response, hybrid aligned cells have maximum measured phase of about half that of planar cells.

The ability to analyse the phase modulation properties in the Fourier plane of liquid crystal devices is a significant insight. For example, we have used such information from this technique to demonstrate improved simulated annealing of phase holograms on a twisted nematic liquid crystal television originally designed for intensity modulation.

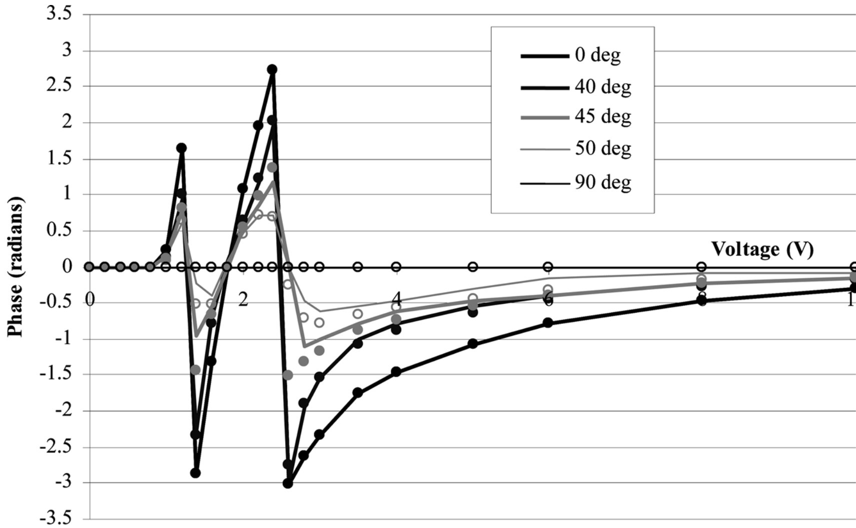

FIGURE 5 Measured and calculated phase for parallel aligned, planar nematic cell. Lines indicate measured results, and marked points, calculations. Labels indicate angle between input polarisation and rubbing direction.

6. DYNAMIC RESULTS AND DISCUSSION

The rise time measured (Fig. 6) decreases with applied voltage. It is found that hybrid aligned cells have a faster rise time than parallel aligned cells. There is little advantage in using an alignment layer that has a higher pre-tilt of 8 degrees. The rise time at an applied voltage of 3 V is 7 ms for a hybrid cell and 20 ms for a parallel alignment cell. The minimum rise time measured was 0.9 ms.

The fall time (Fig. 7) for each cell is approximately constant whatever the applied voltage, and for most cells the fall time was measured to be around 70 ms. The fall time is significantly larger than the best or average rise time. The pi-cell has lowest fall time, although compared to other cells in this study the difference is small. Equations given in literature give the fall time (14) calculated as 76.8 ms, which agrees well with measurements. If the constant of proportionality is the same for rise (12) and fall times (13) the curve that results for fall time is that drawn for hybrid cells (Fig. 7). Therefore the cell topology, and reduced flow improves the rise time of a hybrid cell by a factor of two over that of a parallel cell.

The measurements of fall time for a 2π phase change when the start voltage is varied (Fig. 8) show the fall time increases when the start voltage is close to the Freedericks transition threshold. Measurements taken on the

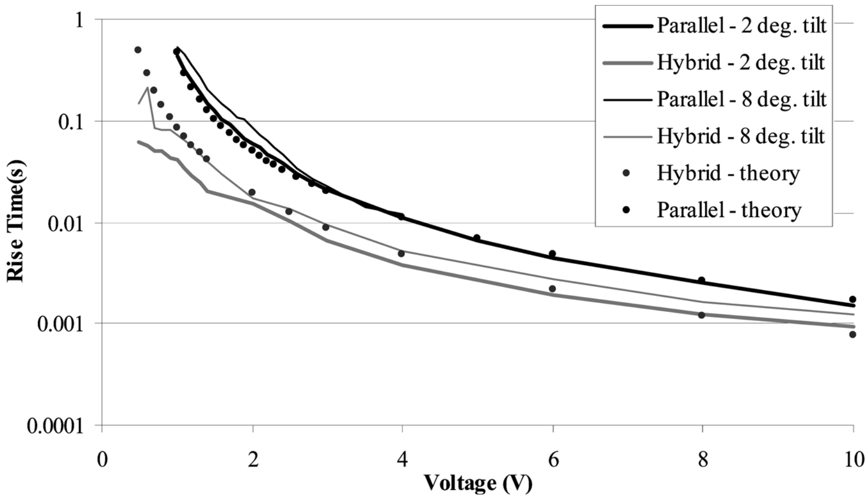

FIGURE 6 Measured rise time of 4 nematic liquid crystal cells.

5 μm cell only showed slight improvement when a higher start voltage is used, so the fall time is reduced to two-thirds of its original value. A very similar effect was observed in both the parallel and antiparallel cells, which suggests flow effects are similar in both cells. Unfortunately the start voltage could not be increased further for the 5 μm cells as a phase

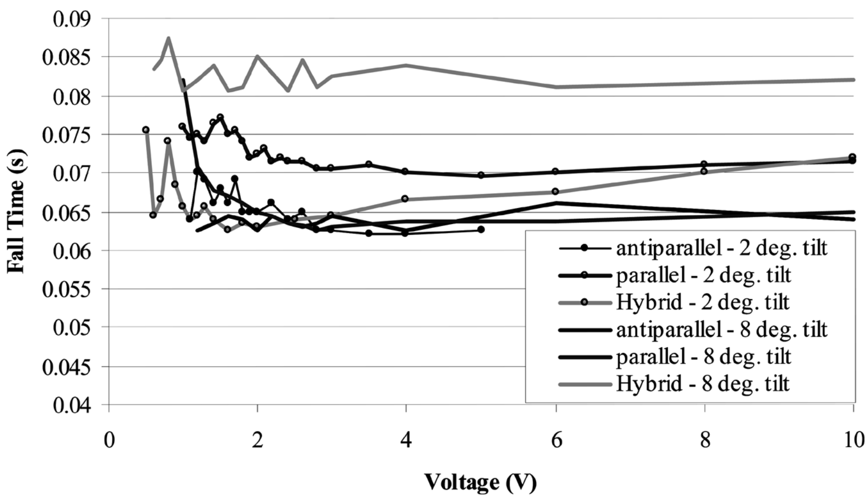

FIGURE 7 Measured fall time of 6 nematic liquid crystal cells.

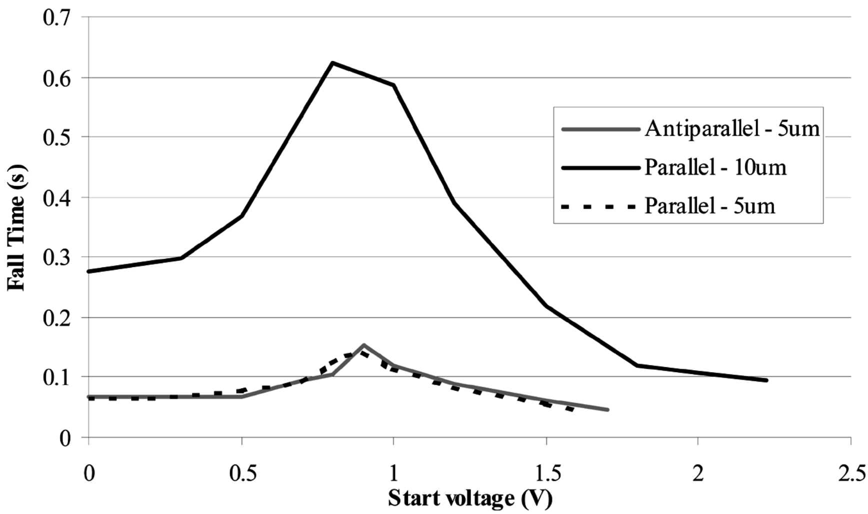

FIGURE 8 Measured fall time of nematic liquid crystal cells for a phase modulation of 2π.

modulation of 2π could not be achieved. At higher voltages it may be that differences become apparent. The thicker cell has greater phase modulation, and the fall time improves to about one third of its original value.

7. CONCLUSIONS

A novel measurement technique has been used to measure phase response time and polarisation dependence of liquid crystal cells. Static measurements show good agreement with theory. A nematic liquid crystal cell exhibits a phase change that decreases as the angle between the input polarisation and liquid crystal director is increased. Dynamic response times are evaluated for nematic cells, and show trends predicted by theory. The rise time of hybrid aligned cells is shorter than that of planar cells, and all cells have a fall time that is approximately constant. Discrepancies are attributed to terms such as pre-tilt and flow that are not considered in this analysis. For the cells measured in this study, no significant advantage was found in increasing the pre-tilt from 2 to 8 degrees.

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